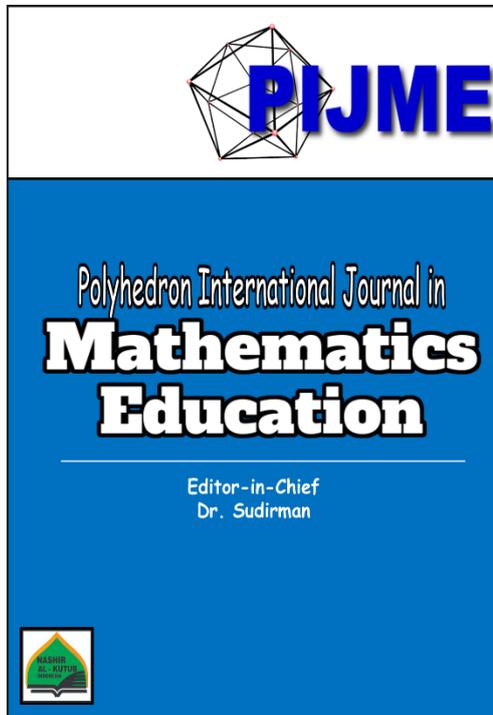


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Ways of Thinking 3D Geometry: Exploratory Case Study in Junior High School Students

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Original Article

Ways of thinking 3D geometry: exploratory case study in junior high school students

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Abstract

Each student has their own characteristics and way of doing 3D geometric thinking. The way of thinking that students do influences the resulting understanding of the concept of 3D geometry. Therefore, this study aims to investigate students' geometric thinking based on the level of achievement of students in completing the 3D geometric thinking ability test (3D GTA). This study uses an exploratory case study design. The participants who voluntarily participated were 33 junior high school students (14 boys, 19 girls) in one of the schools in Indramayu Regency, Indonesia. Data obtained from the process of observation, tests, interviews, and documentation were analyzed qualitatively using Atlas.ti 8 software. The findings revealed that students with low 3D GTA achievements experienced difficulties in representing and calculating the surface area and volume of 3D shapes. In addition, students with moderate 3D GTA achievements experienced difficulties in representing 3D shapes but were able to translate 2D shapes from 3D shapes. Furthermore, students with high 3D GTA achievements experienced difficulties in calculating the surface area and volume of 3D shapes, but were able to use appropriate formulas and were able to interpret the comparisons of 3D geometric shapes well. The results of this study have implications for helping teachers identify student characteristics in understanding the concept of 3D geometry and connections with 2D geometry.

Keywords: 3D geometry way of thinking, 3D geometry concepts, junior high school students

Introduction

Geometry is an important subject in the school mathematics curriculum around the world, which studies shapes, geometric objects, and spatial relationships between objects (Moru et al., 2020; Rodríguez-Nieto, 2021). Students' understanding of geometry will be useful in analyzing, comparing, and generalizing, solving problems both in mathematics and in real-world situations (Lee & Chen, 2014). Jayathirtha (2018) explains that there are three cognitive processes in carrying out 3D geometric thinking processes, namely visualization, construction, and reasoning. In addition, Pittalis et al. (2010) explained that there are five aspects in the 3D geometric thinking process, namely: (1) the ability to represent 3D objects; (2) the ability to recognize and build nets; (3) the ability to

arrange 3D cube layouts; (4) the ability to recognize 3D shape properties and compare 3D shapes; (5) the ability to calculate the volume and area of 3D geometry.

At the school level, mastery of geometric concepts will help students understand other concepts, such as measurement, algebra, calculus and trigonometry (Moru et al., 2020). Herbst et al. (2017) think that studying geometry will help students develop their capacity to organize, predict, and control the world of representations of physical objects and experiences. Therefore, it is natural that geometry is studied at all levels of school, starting from the elementary and secondary levels (Marchis, 2012; Yanik, 2011). Even though geometry is an important concept in mathematics, however, many students have difficulty understanding geometric concepts (Noto et al., 2019).

According to Tock et al. (2020), around 90% of students in their study did not get the maximum score when describing a rectangle from a frontal 3D shape. Often high school students are rarely able to see the cube structure in three-dimensional layers (Hong & Runnalls, 2020). This is also supported by the results of Chiphambo & Mtsi (2021) study, which revealed that students made mistakes when asked to distinguish 2D shapes from 3D shapes, calculating the surface area and total volume of 3D shapes. In addition, according to Parzys (1988) students also have difficulties in representing geometric objects properly. When “seeing” a picture, students tend to think of the properties of the image as the properties of the geometric object itself (Parzys, 1988). Cohen & Ben Gurion (2003) in their research identified five types of student errors in constructing nets from 2D to 3D representation modes. The five types of student errors arise because the net construction process requires students' ability to transform 3D objects into 2D nets by focusing on the parts of the object elements in both representation modes (Pittalis et al, 2010) and requires the ability to manipulating the image (Cohen & Ben Gurion, 2003).

Although there are many researchers who conduct studies on geometric thinking processes, only a few conduct studies on students' 3D geometric thinking processes in junior high schools. Therefore, it is necessary to carry out studies related to the 3D geometric way of thinking. According to Jayathirtha (2018), the geometrical concepts studied by high school students include three different and interrelated cognitive processes, namely visualization, construction, and reasoning. Therefore, an assessment of students' thinking processes can reduce learning obstacles (problems) that occur in learning mathematics, especially in 3D geometry material. The formulation of the research problem is:

- 1) How do students think about representing 3D geometric shapes?
- 2) How do students think about recognizing and constructing 3D geometric shape networks?
- 3) How do students think about translating 2D geometric shapes from 3D geometric shapes?
- 4) How do students think about calculating the number of unit cubes from 3D geometric shapes?
- 5) How do students think about calculating the surface area and volume of 3D geometric shapes?
- 6) How do students think about comparing 3D geometric shapes?

Research Methods

Research design

The study uses an exploratory case study design, which aims to explore the 3D geometric thinking of junior high school students in (1) identifying 3D geometric objects; (2) constructing 3D geometry object nets; (3) translating 2D representation modes from 3D shapes; (4) counting the number of unit cubes; (5) calculate the surface area and volume of 3D geometry; (6) comparing 3D geometric shapes based on their properties. Furthermore, (Sudirman & Martadiputra, [2020](#)) explains that exploratory case studies can be used to explore the difficulties students experience in understanding mathematics, especially 3D geometry material.

The stages of the research carried out in this study began with observing the school to find out the initial conditions related to the research focus, then the researcher conducted a literature review according to the research sub-focus using a qualitative approach strategy. Furthermore, researchers determine participants and make guidelines for observations, tests, interviews, and documentation according to the data needed. After that, the researcher collected data and analyzed the research data with data reduction using the Atlas. ti 8 software and presented the conclusions of the study.

Research Participants

The participants involved in this study came from class VIII students at the junior high school level in Indramayu Regency with a total of 33 students consisting of 14 boys and 19 girls. In practice, before collecting data through tests, the researcher asked permission from the school principal to get a general description of student characteristics. In addition, the researcher also asked permission from students and parents to minimize misunderstandings in conducting this research. After that, the researcher was assisted by the mathematics teacher to choose a class that would be used as the object of research. Next, the researcher gave the 3D KGB test and analyzed the students' answers. After analyzing the students' answers, the research selected three students with low, medium, and high test results to find out an in-depth description of the 3D geometry thinking process in answering the 3D KBG test questions. In addition, the purpose of the interviews was also to reveal students' obstacles in the process of representing, recognizing, and building nets, translating, calculating areas and volumes, and comparing 3D geometric shapes.

Data collection

Methods of data collection in this study using observation, tests, interviews, and documentation. Observations were made by observing the 3D geometry teaching process carried out by the mathematics teacher at the school. In addition, the observation also aims to obtain information related to student learning outcomes in learning mathematics, especially 3D geometry.

After the observation process, in the next stage the researcher gave a 3D Geometry Thinking Ability test (GTA 3D) to the students who were the research subjects. Taking this test is done directly on the answer sheet that has been provided. During the process of working on the test, the researcher directly monitored the progress of the completion of the test by the students in the class. Furthermore, after the test taking process, the researcher processed the test results and prepared 3 students with low, medium, and high test achievement levels to be interviewed. Category determination is calculated based on the total maximum score and ideal minimum score obtained by students in working on the GTA 3D test questions.

The interviews in this study aimed to find out the students' thinking processes in depth. Interviews were conducted directly by involving 3 students in stages.

During the process of interviews and other data collection, researchers took pictures and recorded them as documentary evidence which were then analyzed to become interview transcripts.

Data analysis

To analyze the data obtained from the results of observations, tests, interviews, and documentation, researchers used the Atlas. ti 8 software. The process of using Atlas. ti begins with reducing data, presenting data, and verifying results. Data reduction in this study began with data grouping to make it easier for researchers, this data grouping was done by making research field notes and interview transcripts into documents. After that, the researcher carried out an advanced data reduction process with Atlas. ti 8 software with the following steps:

- 1) Open the Atlas. ti 8 software, then select Create New Project and provide the project name that was created (eg the results of an S-1 interview).
- 2) After the display looks like the image below, click add document on the toolbar then select the file to be uploaded (eg S-1 interview transcript), then click open.
- 3) Open the added document, then start reducing the data by marking the paragraphs or each line that you want to reduce, then click open coding and provide the code according to the sub-focus and research focus. Do the coding until all the text or images obtained from the data get the code.
- 4) After the data has a code, then the visualization stage of the coding result image. Click open network then right click adds neighborhoods and click codes.
- 5) Click layout, and choose randomly. Next, categorize coding according to the research sub-focus by removing the checklist on Show Code-Documents Connections on the toolbar
- 6) Shift each coding box by grouping it sequentially according to the sub-focus and indicator data obtained. Then give a different color in each sub-focus. After that, select the part you want to connect, point to the red circle, then point to the direction of the arrow, and select according to the sub-focus we want.
- 7) Take the same steps to reduce the data from the interview results of the next interview subject using Atlas. ti 8 software.

After the data is reduced, the next step is to present the data from the conclusions. The last stage in this study is the stage of verifying the results of data collection, namely the process of compiling a complete report and presenting the conclusions obtained according to the focus and sub-focus of this study.

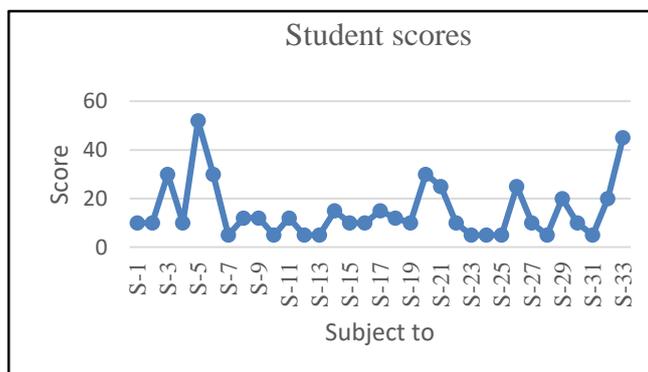
Findings and Discussion

Findings Based on 3D Geometry Thinking Ability Test Results

Based on the analysis of the achievement of the 3D GTA test results, various values were obtained. The distribution of student scores can be seen in Figure 1.

Figure 1

Achievement of 3D GTA test results



Based on Figure 1, there were 33 students taking the test to find out the level of achievement of the 3D GTA, and the average score of students in completing the test was 15 with an ideal maximum score of 100. Furthermore, the researcher categorized the level of achievement of students in answering the 3D GTA test questions into 3 levels, namely: 1) the level of achievement of 3D GTA is low (range 0-16); 2) Moderate 3D GTA achievement level (range 17-32); and 3) high level of achievement of 3D GTA (range 33-52). The presentation of descriptive statistical data on student scores can be seen in Table 1 below.

Table 1

Descriptive statistics of Student 3D GTA scores

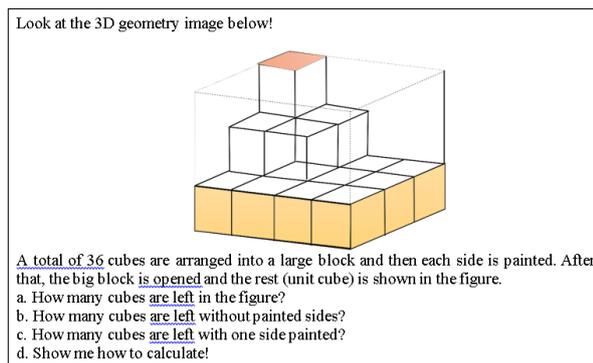
Statistics	Score
Minimum	5
Maximum	52
Range	47
Variant	132
Standard deviation	11

Based on Table 1, it can be seen that the smallest score obtained by students is 5 and the highest score obtained by students is 52 with a range of 47. If seen from the standard deviation value of the data obtained, namely 11 and variant 132, this shows the level of spread of data that is getting farther from the average minimum class passing criteria is 75. Therefore, it can be assumed that students experience difficulties in constructing 3D geometric thinking, resulting in low achievement in 3D KBG test results. Furthermore, the researcher analyzed students' answers in completing the 3D KGB test on each item.

Based on the results of the analysis of student answers to question number 1 (Figure 2), it shows that on average students do not have a good enough ability to represent 3D geometric objects and do not calculate the number of unit cubes carefully. This is shown from the results of the students' answers which did not accurately count the number of cubes with one side painted, as well as the number of cubes whose sides were not painted. This was also supported by the results of teacher interviews which stated that grade 8 students tended to experience difficulties in understanding geometric literacy, especially understanding the shape of word problems related to 3D geometry. This certainly affects the students' thinking processes in constructing 3D geometry problems. Students are hampered to represent 3D geometric shapes, so they have difficulty solving calculation problems related to 3D geometry.

Figure 2

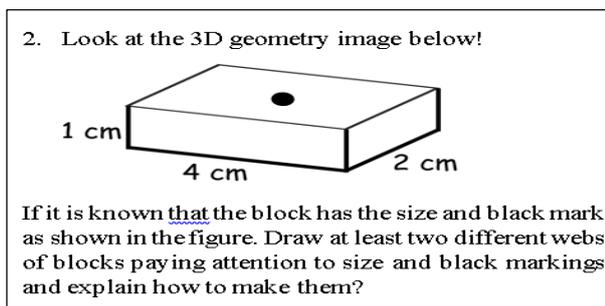
Question number 1



Furthermore, the average results of student answers to question number 2 (Figure 3), namely drawing nets of 3D geometric shapes, showed quite diverse results. The students' answers varied and were still not quite right, which could be caused by a lack of students' understanding of techniques for drawing geometric shapes, both projected (producing 2D images) and isometric (producing 3D geometric shapes). Meanwhile, the results of interviews with mathematics teachers stated that students' lack of ability to re-represent 3D geometric shapes was also due to students' lack of understanding of basic geometric materials. This causes the low ability of students to reason about 3D geometric shapes and elements.

Figure 3

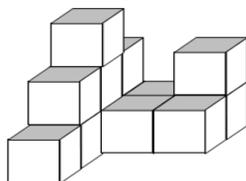
Question Number 2



Next, based on the results of students' answers to question number 3 (Figure 4), which is to re-illustrate geometric shapes in a projection from a 3D figure, the results show that the average student's answer is not correct in the drawing. This can be caused by many factors, both internal factors and external factors that affect the results of student answers. This was also conveyed by the mathematics teacher who said that students had difficulties in representing 3D geometric shapes so students tended to be unsure about drawing 3D geometric shapes that looked sideways and needed the help of measuring tools such as rulers, compasses, and other measuring instruments. Furthermore, the results of student answers in number 3, namely determining the number of high-rise buildings from a 3D geometric shape, showed quite good results. This can be seen from the results of the answers of many students who answered correctly and correctly. Therefore, when viewed from the results of students' answers in number 3, the researcher concluded that the student's ability to re-describe geometric shapes in terms of projections from a 3D spatial figure has a fairly good ability. This is in accordance with the developmental conditions of students who should have been able to describe 3D geometry projected or isometrically.

Figure 4
Question Number 3

3. Look at the 3D geometry image below!

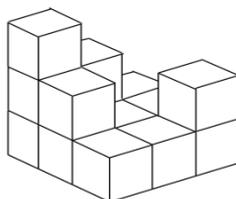


Suppose the pile of unit cubes above is a representation of a multi-storey building. Draw the pile of unit cubes from three directions (front, side and top view)!

Furthermore, based on the results of the average student's answers in solving questions number 4 (Figure 5), namely determining the number of unit cubes from 3D geometric shapes, students have fairly good geometry abilities. This can be seen from the results of students' answers that accurately calculate the number of levels and rooms from the 3D geometric unit cube shape. However, the average student cannot read the questions carefully so the calculation results for each number of rooms are not multiplied by the number of rooms listed in the problem. This suggests that students experience obstacles in calculating unit cubes of 3D geometric shapes.

Figure 5
Questions Number 4

4. Look at the 3D geometry image below!



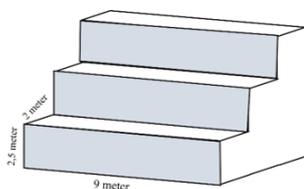
Suppose the pile of unit cubes above represents multi-storey buildings and one tile (cube) contains four rooms of the same shape and size.

- What is the level of the building?
- How many rooms on each level?
- How many rooms are there in the building?
- Show me how to calculate!

Furthermore, in questions number 5 (Figure 6), namely calculating the surface area and volume of the 3D geometric shape of the block. The average student did not calculate correctly and some others did not answer the questions. Furthermore, teacher interviews also stated that students tend to have difficulty in calculating and determining the formulas used when solving problems related to 3D geometry. This then underlies the need for conducting research related to students' thinking processes in constructing 3D geometry, according to the student's internal and external conditions as conveyed by the student's mathematics teacher.

Figure 6
Question Number 5

5. Look at the picture below!



If the picture above represents the stairs of a building. Determine the surface area and volume of the ladder!

Furthermore, from the results of the researcher's interview with the teacher, the researcher obtained data in the form of information related to student's comprehension processes in studying 3D geometry, student learning processes, especially in studying mathematics at school, to obtain responses from teachers regarding the importance of research related to 3D geometry.

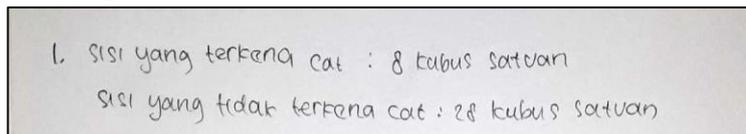
Furthermore, to explore students' thinking processes in completing the 3D GTA test using interviews. Both data from interviews with teachers and students were processed using Atlas software. ti 8. Furthermore, after processing the results of teacher interviews and student 3D GTA tests as a whole, the researcher further selected 3 interview subjects to determine the level of 3D GTA students based on low, medium, and high achievement levels. The interview subject chosen was a student from each GTA 3D achievement level. Next, the researcher processed the data from the results of student interviews with the achievement level of the 3D GTA using the Atlas. ti 8 software. The results of the test analysis were related to students' thinking processes which were indicated in the formulation of the problem in this study, namely the process of representing 3D geometric shapes, recognizing, and building nets 3D geometric shapes, translate 2D geometric shapes from 3D geometric shapes, calculate the number of unit cubes from 3D geometric shapes, calculate the surface area and volume of 3D geometric shapes, and compare 3D geometric shapes.

Findings in representing 3D geometric shapes

The research findings based on the results of interviews with the first subject students with a low level of achievement on the test results (S-1), stated that students' thinking processes in representing 3D geometric shapes experienced obstacles in solving them. This can be seen from the results of student answers in Figure 7.

Figure 7

Answer S-1 Number 1



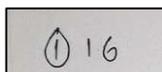
Based on Figure 3, it can be seen that the results of the students' answers stated that they were able to interpret the questions correctly. However, students did not answer correctly the results of calculating the number of unit cubes. This is also supported by the results of student interviews which state that students can represent

3D geometric shapes but have many obstacles. Therefore, it can be concluded that students have difficulties using the basic operating concepts of multiplication and addition to representing 3D geometric shapes.

Furthermore, based on the results of the tests that were given to the second subject students with moderate achievement levels (S-2), the students' thinking processes were obtained in representing 3D geometric shapes as depicted in Figure 8 following.

Figure 8.

Answer S-2 Number 2

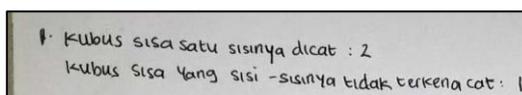


Based on Figure 8, the results of student answers in determining the number of unit cubes representing 3D geometric shapes with one side exposed to paint and the sides not exposed to paint are 16. This is also supported by the results of interviews which state that students have difficulty categorizing answers specifically, which includes the number of unit cubes whose sides are painted on or the remaining cubes whose sides are not painted. Therefore, it can be concluded that students' thinking processes in representing 3D geometric shapes have difficulties.

Furthermore, the student's thinking processes in representing 3D geometric shapes based on the test results of the third subject students, namely students with high 3D GTA achievements (S-3), obtained the results illustrated in Figure 9.

Figure 9

Answer S-3 Number 1



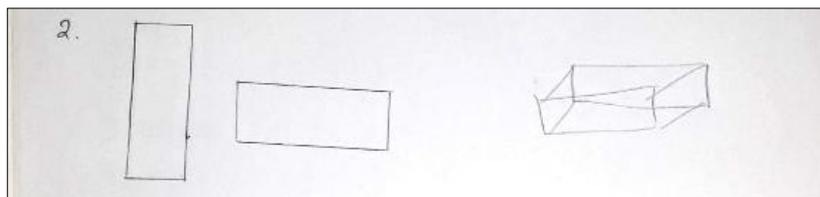
Based on Figure 9, students with high 3D GTA achievement levels have been able to categorize their answers in determining the number of remaining cubes with one side covered with paint and the remaining cubes whose sides are not stained. In addition, the results of student calculations in calculating the number of cubes whose sides are not exposed to paint are also correct and correct. In addition, the results of the interviews stated that the students had difficulties and confusion in determining the number of unit cubes with one side exposed to paint and unit cubes whose sides were not stained. Therefore, researchers can conclude that students have been able to complete the test well, it's just that students feel insecure about the results of their answers so they require a confirmation from the teacher regarding their thought processes.

Findings in constructing 3D Geometry Nets

Based on the test results of the first subject students, namely students with low 3D GTA achievements (S-1), it is known that the students' thinking processes in recognizing and building 3D geometric shape nets can be seen in Figure 10.

Figure 10

Answer S-1 Number 2

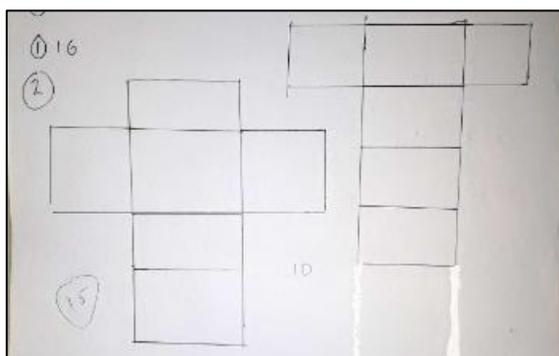


Based on the results of students' answers in Figure 10, shows that students can only describe 2 rectangular shapes with different positions. Furthermore, based on the interview results, students' thinking processes in recognizing and constructing 3D geometric shape nets obtained the result that students stated that it was easy to describe 3D geometric network shapes, especially the blocks asked for in the problem. Therefore, it can be concluded that students already know the concept of 3D spatial nets and know and are easy to build 3D nets.

Furthermore, the student's thinking processes in recognizing and constructing 3D geometric shape nets based on the test results of students with moderate 3D GTA achievement levels (S-2), can be seen from the results of student answers in Figure 11.

Figure 11

Answer S-2 Number 2

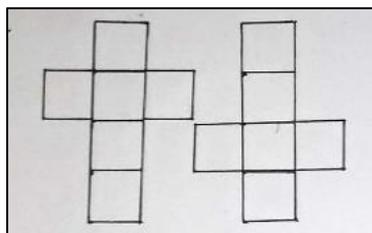


Based on Figure 11, it can be seen that the results of the students' answers describe the nets and their sizes match the beam nets. In addition, students can also describe more than one type of net with different shapes. However, this was not supported by the results of the student interviews. The interview results stated that students found it difficult to answer question number 2. This was because students did not have knowledge regarding the concept of beam nets and students had difficulty drawing beam nets. Therefore, it is better for the teacher as a facilitator in terms of learning to provide knowledge to students about the concept of nets from basic 3D geometric shapes further.

Next, the student's thinking processes in recognizing and building 3D geometric shape nets when seen from the test results of the three interview subjects with high 3D GTA achievements (S-3), can be seen in Figure 12.

Figure 12

Answer S-3 Number 2



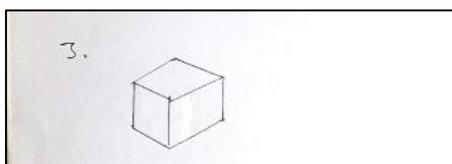
Based on the results of the students' answers from Figure 12, it can be seen that students are already able to describe the shape of the nets in a 3D shape. However, the students did not accurately describe the length and width of the beam nets. Next, the results of student interviews stated that it was easy to describe the shapes of 3D geometric nets according to their size and had no difficulty in understanding the concept of 3D shape nets. Thus, students with moderate thinking skills are able to recognize and construct 3D geometric shape net.

Findings in Translating 2D Shape Representation Modes from 3D Geometry Shapes

In question number 3, the thinking process of students in translating 2D geometric shapes from 3D geometric shapes based on the test results of the first subject students with low 3D GTA test results (S-1), can be seen in Figure 13..

Figure 13

Answer S-1 Number 3

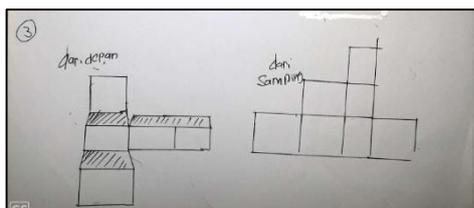


Based on Figure 13, it can be seen that the results of students' answers were unable to draw using the projection technique (producing 2D images). Thus, it can be concluded that students with low thinking skills do not have knowledge related to the concept of drawing in a projection or isometric manner (producing 3D geometric images). In addition, the results of student interviews stated that students had difficulty answering question number 3. Therefore, it can be concluded that students cannot analyze the questions properly so they fail to describe the 2D shapes that appear in the front, side view, or top view of existing 3D shapes.

Based on Figure 14, it can be seen that students with 3D GTA achievements are not accurately depicting 2D geometric shapes from the projected images.

Figure 14

Answer S-2 Number 3

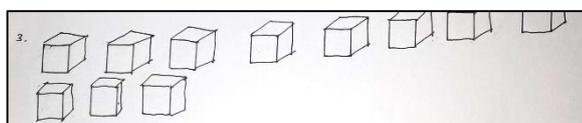


Furthermore, the student's thinking process in translating 2D geometric shapes from 3D geometric shapes is concluded from the results of the Master's interview, that is, students cannot solve problem number 3 easily. This is following the results of the reduction in student interviews in Figure 14 which states that students have difficulty describing 3D shapes into 2D shapes from the front, side, and top views. Therefore, it can be concluded that students with moderate achievement in the 3D GTA test results (S-2) cannot translate 2D geometric shapes from 3D geometric shapes properly.

Furthermore, the thinking process of students in translating 2D geometric shapes from 3D geometric shapes of students with high 3D GTA test results (S-3) can be seen in Figure 15.

Figure 15

Answer S-3 Number 3



Based on Figure 15, it can be seen that the results of the students' drawings did not accurately describe the shape of the image in terms of projection from the front view, side view, and top view of a 3D spatial figure. It can be assumed that in addition to students not knowing the concept of drawing in a projected way, students are also unable to answer questions correctly. Furthermore, the results of the reduction in student interviews stated that students were able to translate 2D geometric shapes from 3D geometric shapes. Therefore, students have difficulties and obstacles in translating 2D geometric shapes from 3D geometric shapes.

Research Results Calculating the Number of Unit Cubes from 3D Geometry Shapes

The findings of the research on students' thinking processes in calculating the number of unit cubes of 3D geometric shapes can be seen from the results of students' answers in solving test questions numbers 4. As for the students' thinking processes in calculating the number of unit cubes of 3D geometric shapes from the first student as interview subjects with the achievement level of 3D KGB low (S-1), states that students have difficulties so they cannot solve the problem correctly. The students' thinking processes can be seen in Figure 16.

Figure 16

Answer S-1 Numbers 4

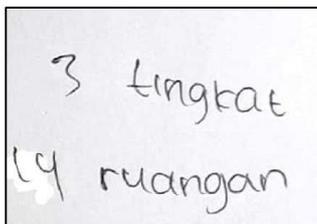


Figure 16 results of students' answers being able to answer the exact number of levels of the building requested from the available 3D geometric shapes. Next, when viewing the results of the interviews, students found it difficult and could not

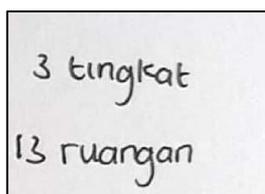
imagine 3D geometric shapes. Meanwhile, in processing geometric thinking calculating the surface area and volume of 3D geometric shapes students stated that they found it difficult and did not know the formulas for the surface area of a cuboid. Therefore, it can be concluded that in calculating the number of unit cubes of 3D geometric shapes students must increase the accuracy and process of a more detailed analysis.

Meanwhile, the student's thinking processes in calculating the number of cubes of 3D geometric shapes by the second interview subject (S-2) in this study, the student's answers were able to calculate the exact number of levels contained in the image. This was supported by the results of the student interviews stating that the process of determining the number of stories in the building is easy and well-represented. Meanwhile, in calculating the number of rooms in the building, students have obstacles in the form of difficulty in calculating the exact number of rooms.

Furthermore, the student's thinking processes in calculating the number of unit cubes of 3D geometric shapes from the test results of students with high 3D KGB achievements (S-3), can be seen in Figure 17..

Figure 17

Answer S-3 Numbers 4



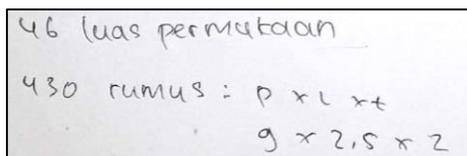
Based on Figure 13, it was found that the students' thinking processes in calculating the number of unit cubes of 3D geometric shapes by the third interview subject were appropriate and appropriate. Therefore, this assumes that students have been able to interpret the questions correctly, it's just that students don't calculate correctly.

Findings in Calculating Surface Areas and Volumes of 3D Geometry Shapes

The findings of the research on students' thinking processes in calculating the surface area and volume of 3D geometric shapes can be seen from the results of students' answers in solving test questions number 5. The students' thinking processes in calculating the surface area and volume of 3D geometric shapes of students with low 3D GTA achievements (S- 1), can be seen from the results of student answers in Figure 18.

Figure 18

Answer S-1 Numbers 5



Based on Figure 18, it can be seen that students did not use the correct formula in calculating the surface area of a block. In addition, based on the results of interviews the students stated that they could not solve calculation problems related to 3D

geometry. Thus, it can be concluded that students do not know the concept of calculating the surface area of a 3D shape and cannot use the correct formula in completing the calculation.

Furthermore, the findings of research on students' thinking processes in calculating the surface area and volume of 3D geometric shapes of students with moderate 3D GTA achievements (S-2), can be seen from the results of student answers in Figure 19.

Figure 19

Answer S-3 Numbers 5

The image shows handwritten calculations for a rectangular prism with dimensions 9x2x5. The student calculates the surface area as $L = P \times L = 9 \times 2 = 18$ and the volume as $V = P \times L \times t = 9 \times 2 \times 5 = 45,0$. For the surface area, they use the formula $L_1 + L_2 + L_3 = 18 + 18 + 18 = 54$. There are some additional scribbles and a '5' written next to the volume calculation.

Based on Figure 19, shows that students have been able to interpret the questions correctly, it's just that students don't use the formula for the surface area of a block correctly. Meanwhile, based on the results of student interviews with a moderate level of achievement in the 3D GTA test results (S-2), stated that when carrying out the thinking process in calculating the surface area and volume of 3D geometric shapes students are hampered. Therefore, students with the ability level of geometric thinking The high level in this study does not require more accuracy and knowledge regarding the surface area formulas for 3D shapes, especially beams.

Meanwhile, the results of the third interview subject's answers at number 7 were appropriate in solving the volume of blocks with the right formula. Furthermore, this is supported by the results of student interviews with high levels of 3D GTA achievement (S-3) from the reduction results with Atlas. ti 8 software, the results obtained are that students do not know the formula for the surface area of a beam and the formula for the volume of a beam. it was concluded that the students already had a fairly good ability in solving 3D geometry problems to find the surface area and volume.

Discussion

Based on the findings with the help of Atlas. ti 8 shows that the thinking process of 3D geometry in students who have low to high ability levels is that students cannot represent 3D geometric shapes properly. The findings of this study are also in line with those presented by Stylianou (2011), who explained that identifying 3D geometric shapes is part of the process of representing 3D shapes, which is a very complex process, because it involves concepts and properties of 2D objects or 2D space representations. Images, which are used in representing 3D geometric shapes, are part of the 2D representation space. The process of connecting the two spaces has many didactic implications in understanding the concept of 3D geometry (Mesquita, 1998). In addition, the two spaces have very different properties, a mathematician usually works through a geometric space, while students work in a representational space (Mesquita, 1998).

Even though representing is a process that is difficult for students to understand, according to Fonna & Mursalin (2019), explaining that the ability to

represent is the basis of the ability to develop concepts and think mathematically. According to Zheng et al. (2021), representing 3D objects effectively and efficiently for junior high school students is essential for many processes in other fields. Therefore, in representing 3D geometry students are required to be able to integrate the skills of 3D model reconstruction, matching, manipulation, and problem understanding. However, the unavailability of adequate learning media makes it difficult for students to understand geometric concepts clearly.

In addition to difficulties in identifying the elements of 3D geometric shapes (points of corners, edges, and planes), students also have difficulties in drawing nets from 3D geometric shapes. The findings of this study are in line with Cohen & Ben Gurion (2003) in their research identifying five types of student errors in constructing nets from 2D to 3D objects, namely: (1) confusion between the perspective view of the solid and its net (this type is caused by students experiencing confusion in seeing 3D geometric shapes from various perspectives to the construction of the nets); (2) Joining the disc and the lateral surface along a line (this type is related to how the various types of ribs are arranged in the specified 2D plane); (3) Wrong form of the edge to be joined (this type is caused because students are wrong in combining the edges of the fields that are formed); (4) Wrong placement of the parts (this type is related to the student's failure to imagine the parts must be connected to each other) (5) Other mistakes (this type is caused because the student does not have ideas in constructing nets). The five types of student errors are due to the construction of nets from 3D shapes requiring the ability of students to transform 3D objects into 2D nets by focusing on studying the component parts of objects in both representation modes (Pittalis et al., 2010a) and requires the ability to manipulating the image (Cohen & Ben Gurion, 2003).

According to Wu & Song (2015), building a 3D shape net can be started by recognizing object categories, completing 3D shapes, and estimating the best view into a suitable image. In addition, according to Ocal & Halmatov (2021) direct teaching by the teacher will help students identify 3D shape nets.

Furthermore, the results of this study also show that students experience difficulties in translating parts of 2D geometric shapes that are generated from 3D geometric shapes. The difficulty is caused by the inability to determine the arrangement of the ribs on the front, side and top. The average student thinking process in translating 2D geometric shapes from 3D geometric shapes shows that students with low to high ability levels have difficulty interpreting the meaning of the questions and cannot easily imagine the geometric shapes in the questions.

The findings of this study are in line with the findings of Fujita et al. (2020), who concluded that there were five student errors in translating 3D geometric images based on their properties, namely (1) type 1, wrong answer because they only used intuition to answer questions without the use of spatial visualization and spatial analytical reasoning based on their properties explicitly; (2) type 2A, wrong answer because it only uses spatial analytic reasoning based on its properties but is influenced by visual information and/or incorrect/inappropriate material knowledge; (3) type 2B, the answer is wrong because it only uses the spatial visualization process but is influenced by visual information and/or incorrect/inappropriate material knowledge; (4) type 2C, the answer is wrong because it uses spatial visualization and spatial analytic reasoning based on its properties but is influenced by visual information and/or wrong/inaccurate

knowledge of the material; (5) type 3, correct answer using valid spatial visualization and/or spatial analytic reasoning based on its properties with correct material knowledge. The results of this study are also in line with the research by Tock et al. (2020), most students did not get the maximum score when describing a rectangle from a 3D front view. There were only 10% of students who succeeded in depicting 3D objects according to the correct viewpoint. This shows that to improve student learning outcomes, adequate retention facilitation is needed regarding techniques for drawing 3D geometric shapes both manually and using digital applications (Ng et al., 2020).

According to Yildiz & Özdemir (2021), building a 2D geometric thinking structure that comes from the representation of 3D shapes requires various types of reasoning regarding the correct representation of 3D geometric concepts. In addition, to be able to integrate 3D spatial displays into understandable forms, a significant correlation is needed between spatial abilities and the ability to draw 3D shapes (Lee & Lee, 2019). Therefore, translating 3D geometric shapes into 2D shapes or vice versa requires a basic understanding and reasoning about the differences between the two modes of representation. In addition, according to Fujita et al. (2020) stated that solving problems involving 2D representations of 3D geometric shapes plays an important role in building students' spatial reasoning skills.

Furthermore, related to the thought process in determining the number of unit cubes, it was found that some students were able to calculate the number of unit cubes of 3D geometric shapes properly, it's just that there were also students who were unable to understand the problem information accurately and correctly. Therefore, students need to improve their accuracy and visual ability in interpreting the questions.

The results of this study are also in line with his research, Chairunnisa *et al.* (2021) concluded that students with a low level of spatial ability (51.4%) experienced difficulties from Level 0 (visualization), Level 1 (analysis), Level 2 (informal deduction), Level 3 (reduction), and Level 4 (rigor). The same thing was also revealed based on the research results of Wahab *et al.* (2016) identified weaknesses in visual spatial skills among outstanding students, for example mental cut, mental rotation, and mental view.

The findings of this study are in line with the opinion of Battista & Clements (1996), who explained that the development of spatial structure is not a simple procedure, because most students in secondary schools have difficulty doing tasks related to spatial structure. For example, when students were asked to construct a cube with five-unit edges, most students had difficulty determining the unit cubes needed to cover the base, which was a "unit/layer" (Battista & Clements, 1996). This difficulty is because students do not have clear procedures and stages (Mulligan et al., 2020).

Furthermore, related to students' thinking processes in the average student thinking process in calculating the surface area and volume of 3D geometric shapes, it can be concluded that students already know the concept of calculating the surface area and volume of 3D geometric shapes, but students need further understanding and accuracy in calculating them. In addition, students also need to improve the process of representing 3D geometric shapes correctly and precisely so that they can solve problems correctly.

The findings of this study are in line with Chiphambo & Mtsi (2021) the study revealed that 61% of students could not distinguish 2-Dimensions from 3-Dimensions; 33% of students were wrong in calculating the total surface area, 44% of students were wrong in using the prism volume formula; and 33% of students misunderstood mathematical terms related to the surface area of a prism. Tanisman & Aksu (2016) revealed that there were misconceptions and errors when students measured length, surface area, and volume. Kusumaningrum *et al.*, (2020) concluded that the difficulties faced by students in answering flat-sided geometrical material were: (a) students had many difficulties in understanding the basic geometric concepts; (b) students still have difficulty understanding the questions so they have difficulty in determining the formula to be used; (c) lack of good analysis of orders from questions.

Conclusion

Based on the results of observations, tests, and interviews with students and teachers, the student's thinking processes in constructing 3D geometry obtained the result that the students' thinking processes in representing 3D geometric shapes Grade 8 had difficulty determining the number of unit cubes, both one side exposed to paint and one cube units whose sides are not exposed to paint. Meanwhile, students' thinking processes in recognizing and building nets of student 3D geometric shapes obtained the result that Grade 8 students were able to build and recognize nets. However, students need further assistance from the teacher to be precise in determining the correct size of 3D spatial networks. In this context, teachers could implement everyday artifacts to foster connections between real-life objects (e.g., cardboard boxes, wooden cubes, etc.) and 3D figures in order to improve understanding (Rodríguez-Nieto, 2021).

Furthermore, students' thinking processes in translating 2D geometric shapes from 3D geometric shapes of grade 8 students require further understanding regarding geometric drawing techniques both in projection (producing 2D images) and isometrically (producing 3D images). Next, the thinking process of grade 8 students in calculating the number of unit cubes from 3D geometric shapes has been able to represent and calculate the number of unit cubes of 3D geometric shapes, the result is that students need to improve their accuracy and accuracy in interpreting the questions.

Furthermore, students' thinking processes in calculating the surface area and volume of 3D geometric shapes can be concluded that students already know the concept of calculating the surface area and volume of 3D geometric shapes but students need further understanding and accuracy in calculating it. Meanwhile, the student's thinking process in comparing 3D geometric shapes was that students did not calculate the exact volume of each 3D shape. This can be caused by several things including students not counting them carefully and students needing further understanding regarding the basic concepts of calculation, both addition, and multiplication.

Implications

The results of the research are useful in contributing ideas or enriching concepts to science. In addition, this research can also be used as reference material for other researchers who wish to conduct similar research. Next, from a practical point of

view, this research provides benefits, among other things, the results of this study are expected to add to the insights of researchers. Apart from that, for teachers, it is hoped that it can be used as an alternative to improve the quality of mathematics education in junior high schools, especially in reducing students' learning obstacles in determining geometric thinking 3D.

Limitation

The problem in this research cannot know the students' way of thinking in constructing 3D geometric thinking.

References

- Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education*, 27(3), 258–292. <https://doi.org/10.2307/749365>
- Chairunnisa, C., Surya, E., & Sinaga, B. (2021). The Difficulty Analysis of Students Spatial Ability Based on Level Van Hiele in Problem Based Learning. *Budapest International Research and Critics in Linguistics and Education*, 4(1), 158–164. <https://doi.org/10.33258/birle.v4i1.1569>
- Chiphambo, S. M., & Mtsi, N. (2021). Exploring Grade 8 Students' Errors When Learning About the Surface Area of Prisms. *EURASIA Journal of Mathematics, Science and Technology Education*, 17(8), em1985.
- Cohen, N., & Ben Gurion. (2003). Curved solids nets. *International Group for the Psychology of Mathematics Education*, (2), 229–236.
- Fonna, M., & Mursalin, M. (2019). Using of Wingeom Software in Geometry Learning to Improving the of Mathematical Representation Ability. *Malikussaleh Journal of Mathematics Learning (MJML)*, 1(2), 40–43. <https://doi.org/10.29103/mjml.v1i2.1174>
- Fujita, T., Kondo, Y., Kumakura, H., Kunimune, S., & Jones, K. (2020). Spatial reasoning skills about 2D representations of 3D geometrical shapes in grades 4 to 9. *Mathematics Education Research Journal*, 32(2), 235–255. <https://doi.org/10.1007/s13394-020-00335-w>
- Herbst, P., Fujita, T., Halverscheid, S., & Weiss, M. (2017). The learning and teaching of geometry in secondary schools: A modeling perspective. In *The Learning and Teaching of Geometry in Secondary Schools: A Modeling Perspective*. <https://doi.org/10.4324/9781315267593>
- Hong, D. S., & Runnalls, C. (2020). Understanding length \times width \times height with modified tasks. *International Journal of Mathematical Education in Science and Technology*, 51(4), 614–625. <https://doi.org/10.1080/0020739X.2019.1583383>
- Jayathirtha, G. (2018). An analysis of the national intended geometry curriculum. *Contemporary Education Dialogue*, 15(2), 143–163. <https://doi.org/10.1177/0973184918783291>
- Kusumaningrum, B., Irfan, M., & Zainnur Wijayanto. (2020). Errors Analysis of Students in Solving Volume of the Solid of Revolution Problem in Term of Critical Thinking Aspects. *Kalamatika: Jurnal Pendidikan Matematika*, 5(2), 119–132. <https://doi.org/10.22236/kalamatika.vol5no2.2020pp119-132>
- Lee, C. Y., & Chen, M. J. (2014). The impacts of virtual manipulatives and prior knowledge on geometry learning performance in junior high school. *Journal*

- of Educational Computing Research*, 50(2), 179–201.
<https://doi.org/10.2190/EC.50.2.b>
- Lee, I., & Lee, I. (2019). Using augmented reality to train students to visualize three-dimensional drawings of mortise – tenon joints in furniture carpentry carpentry. *Interactive Learning Environments*, 0(0), 1–15.
<https://doi.org/10.1080/10494820.2019.1572629>
- Marchis, I. (2012). Preservice Primary School Teachers' Elementary Geometry Knowledge. *Acta Didactica Napocensia*, 5(2), 33–40.
- Mesquita, A. L. (1998). On conceptual obstacles linked with external representation in geometry. *Journal of Mathematical Behavior*, 17(2), 183–195.
[https://doi.org/https://doi.org/10.1016/S0364-0213\(99\)80058-5](https://doi.org/https://doi.org/10.1016/S0364-0213(99)80058-5)
- Moru, E. K., Malebanye, M., Morobe, N., & George, M. J. (2020). A Van Hiele Theory analysis for teaching volume of three-dimensional geometric shapes. *Journal of Research and Advances in Mathematics Education*, 6(1), 17–31.
<https://doi.org/10.23917/jramathedu.v6i1.11744>
- Mulligan, J., Woolcott, G., Mitchelmore, M., Busatto, S., Lai, J., & Davis, B. (2020). Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, 32(2), 285–305. <https://doi.org/10.1007/s13394-020-00324-z>
- Ng, O. L., Shi, L., & Ting, F. (2020). Exploring differences in primary students' geometry learning outcomes in two technology-enhanced environments: dynamic geometry and 3D printing. *International Journal of STEM Education*, 7(1), 1–13. <https://doi.org/10.1186/s40594-020-00244-1>
- Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The Learning Obstacles of Pre-Service Mathematics Teachers on Transformation Geometry. *Journal on Mathematics Education*, 10(1), 117–125.
<https://doi.org/10.22342/jme.10.1.5379.117-126>
- Ocal, T., & Halmatov, M. (2021). 3D geometric thinking skills of preschool children. *International Journal of Curriculum and Instruction*, 13(2), 1508–1526. Retrieved from <http://ijci.wcci-international.org/index.php/IJCI/article/view/404/313>
- Parzysz, B. (1988). "Knowing" vs "Seeing". Problems of the Plane Representation of Space Geometry Figures. *Educational Studies in Mathematics*, 19(1), 79–92.
- Pittalis, M., Mousoulides, N., & Christou, C. (2010a). Students' 3D geometry thinking profiles. *Proceedings of CERME6*, 816–825. Retrieved from <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1074.8742&rep=rep1&type=pdf>
- Rodríguez-Nieto, C. A. (2021). Conexiones etnomatemáticas entre conceptos geométricos en la elaboración de las tortillas de Chilpancingo, México. *Revista de Investigación, Desarrollo e Innovación*, 11(2), 273–296.
<https://doi.org/10.19053/20278306.v11.n2.2021.12756>
- Seah, R. T. K., & Horne, M. (2020). The influence of spatial reasoning on analysing about measurement situations. *Mathematics Education Research Journal*, 32(2), 365–386. <https://doi.org/10.1007/s13394-020-00327-w>
- Stylianou, D. A. (2011). An examination of middle school students' representation practices in mathematical problem solving through the lens of expert work: towards an organizing scheme. *Educational Studies in Mathematics*, 76(3),

- 265–280. <https://doi.org/10.1007/s10649-010-9273-2>
- Sudirman, S., & Martadiputra, B. A. P. (2020). Exploratory Case Study Kesulitan Siswa Sekolah Menengah Pertama dalam Menyelesaikan Permasalahan Luas Permukaan Limas. *Math Didactic: Jurnal Pendidikan Matematika*, 6(3), 277–286.
- Tan-sisman, G., & Aksu, M. (2016). A Study on Sixth Grade Students ' Misconceptions and Errors in Spatial Measurement: Length , Area, and Volume. *International Journal of Science and Mathematics Education*14, 14(7), 1293–1319. <https://doi.org/10.1007/s10763-015-9642-5>
- Wahab, R. A., Abdullah, A. H., Abu, M. S., Moktar, M. B., & Atan, N. A. B. (2016). A case study on visual spatial skills and level of geometric thinking in learning 3D geometry among high achievers. *Man in India*, 92(1), 489–499.
- Wu, Z., & Song, S. (2015). 3D ShapeNets : A Deep Representation for Volumetric Shapes. *IEEE Xplore*, 1–9.
- Yanik, H. B. (2011). Prospective middle school mathematics teachers ' preconceptions of geometric translations. *Educational Studies in Mathematics*, 78, 231–260. <https://doi.org/10.1007/s10649-011-9324-3>
- Yildiz, S. G., & Özdemir, A. S. (2021). The Effects of Engineering Design-Based Mathematics Applications on Spatial Abilities and 3D Geometrical Thinking Skills of Students with Different Learning Approaches. *International Online Journal of Education and Teaching*, 8(2), 1–28.
- Zheng, Z., Yu, T., Dai, Q., & Liu, Y. (2021). Deep Implicit Templates for 3D Shape Representation. *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 4–11. <https://doi.org/10.1109/CVPR46437.2021.00148>